# CIS 7 Chapter 17 Notes: Relations

## Define Relation

A **relation** expresses a connection between the objects of two sets where the two sets may be the same.

**Relations** may exist between objects of the same set or between objects of two or more sets.

A binary relation R from set x to y (written as xRy or R(x,y)) is a subset of the Cartesian product X×Y. If the ordered pair of G is reversed, the relation also changes.

Generally an n-ary relation R between sets A1,…, and An is a subset of the n-ary product A1×⋯×An. The minimum cardinality of a relation R is Zero, 0, and maximum is n2 in this case.

**A binary relation R on a single set A is a subset of A×A.**

For two distinct sets, A and B, having cardinalities **m** and **n** respectively, the maximum cardinality of a relation R from A to B is **mn**.

**Domain:** all x-values that are to be used (independent values).

**Range:** all y-values that are used (dependent values)

If there are two sets A and B, and relation R have order pair (x, y), then –

* The **domain** of R, **Dom(R)**, is the set {x|(x,y)∈ R for some y in B}.
* The **range** of R, **Ran(R)**, is the set {y|(x,y)∈ R for some x in A}

**Example 17.1:**

Let, A = {1,2,9} and B = {1,3,7}

* Case 1 − If relation R is ***'equal to'*** then R={(1,1),(3,3)}
  + Dom(R) = {1,3}, Ran(R) = {1,3}
* Case 2 − If relation R is ***'less than'*** then R = {(1,3),(1,7),(2,3),(2,7)}
  + Dom(R) = {1,2},Ran(R) = {3,7}
* Case 3 − If relation R is ***'greater than'*** then R = {(2,1),(9,1),(9,3),(9,7)}
  + Dom(R) = {2,9}, Ran(R) = {1,3,7}

## Representation of Relations using Graph

A relation can be represented using a directed graph.

Object 1 Object 3

The number of vertices in the graph is equal to the number of elements in the set from which the relation has been defined. For each ordered pair (x, y) in the relation R, there will be a directed edge from the vertex ‘x’ to vertex ‘y’. If there is an ordered pair (x, x), there will be self- loop on vertex ‘x’.

Suppose, there is a relation R={(1,1),(1,2),(3,2)} on set S={1,2,3}, it can be represented by the above graph.

## Types of Relations

1. The **Empty Relation** between sets X and Y, or on E, is the empty set ∅
2. The **Full Relation** between sets X and Y is the set X×Y
3. The **Identity Relation** on set X is the set {(x,x)|x∈X}
4. The **Inverse Relation** R' of a relation R is defined as − R′={(b,a)|(a,b)∈R}

In mathematics, a binary relation R over a set X is reflexive if every element of X is related to itself.[1][2] Formally, this may be written ∀x ∈ X : x R x.

An example of a reflexive **relation is the relation "is equal to" on the set of real numbers**, since every real number is equal to itself. A **reflexive relation** is said to have the reflexive property. Along with symmetry and transitivity, reflexivity is one of three properties defining **equivalence relations**. Clearly, it is true that a = a for all values a. Therefore, a = is reflexive.

**Example 17. 2:**

If R={(1,2),(2,3)} then R′ will be {(2,1),(3,2)}

A relation R on set A is called **Reflexive** if ∀a∈A is related to a (aRa holds)

**Example 17.3:**

The relation R={(a,a),(b,b)} on set X={a,b} is reflexive.

A relation R on set A is called **Irreflexive** if no a∈A is related to a (aRa does not hold).

A **symmetric relation** is a type of binary relation. An example is the relation "is equal to", because **if a = b is true then b = a is also true**.

**Example 17.4:**

The relation R={(a,b),(b,a)} on set X={a,b} is irreflexive.

A relation R on set A is called **Symmetric** if xRy implies yRx, ∀x∈A and ∀y∈A.

**Example 17.5:**

The relation R={(1,2),(2,1),(3,2),(2,3)} on set A={1,2,3} is symmetric.

A relation R on set A is called **Anti-Symmetric** if xRy and yRx implies x=y∀x∈A and ∀y∈A.

**Transitive:** If a = b and b = c, this says that a is the same as b which in turn is the same as c. So a is then the same as c, so a = c, and thus = is transitive.

**Example 17.6:**

The relation R={(x,y)→N|x≤y} is anti-symmetric since x≤y and y≤x implies x=y.

A relation R on set A is called Transitive if xRy and yRz implies xRz,∀x,y,z∈A.

**Example 17.7:**

The relation R={(1,2),(2,3),(1,3)} on set A={1,2,3} is transitive.

A relation is an **Equivalence Relation** if it is ***reflexive, symmetric, and transitive***.

**Example 17.8:**

The relation R={(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)} on set A={1,2,3} is an **equivalence** relation since it is reflexive, symmetric, and transitive.